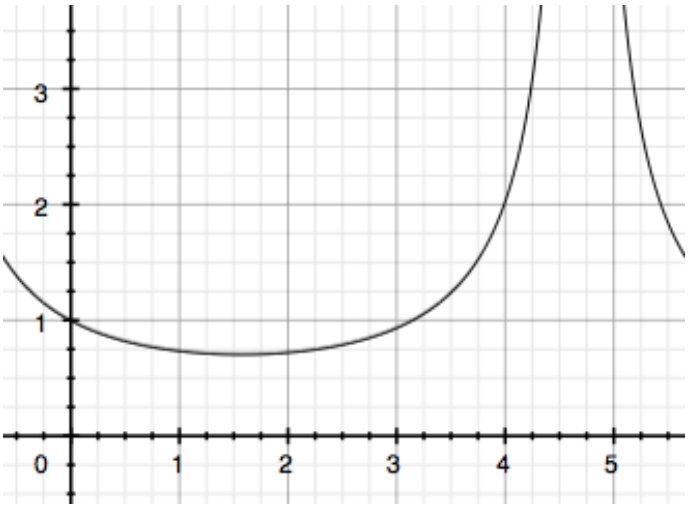
Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series

Numerical Integration - 2

$$f(x) = \frac{1}{\sqrt{1+\sin x}}$$



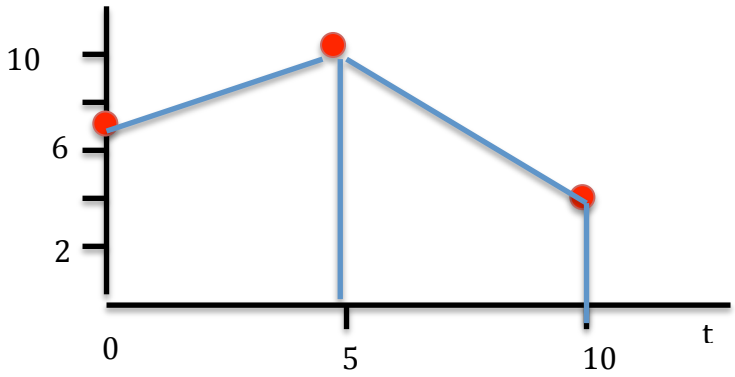
Lets get an approximate value for:

$$\int_{x=1}^{x=4} \frac{1}{\sqrt{1+\sin x}}\,dx$$

Some values of the function $f(x) = \frac{1}{\sqrt{1+\sin x}}$

x	$f(x)$	Weight
1	0.736915277	0.5
1.2	0.719435801	1
1.4	0.70969305	1
1.6	0.70718217	1
1.8	0.71177575	1
2	0.723707722	1
2.2	0.743603072	1
2.4	0.7725606	1
2.6	0.812310062	1
2.8	0.865488486	1
3	0.93612607	1
3.2	1.030530411	1
3.4	1.158989863	1
3.6	1.339323617	1
3.8	1.605109331	1
4	2.027778167	0.5

For three data points:



For two trapezoids the calculation looks like this:

$$5\frac{7+10}{2} + 5\frac{10+4}{2}$$

This is the sum of the areas of the two trapezoids.
Factor out the common interval length:

$$5\left(\frac{7+10}{2} + \frac{10+4}{2}\right)$$

$$5\left(\frac{7}{2}+\frac{10}{2}+\frac{10}{2}+\frac{4}{2}\right)$$

$$5\left(\frac{7}{2}+10+\frac{4}{2}\right)$$

x	$f(x)$	Weight
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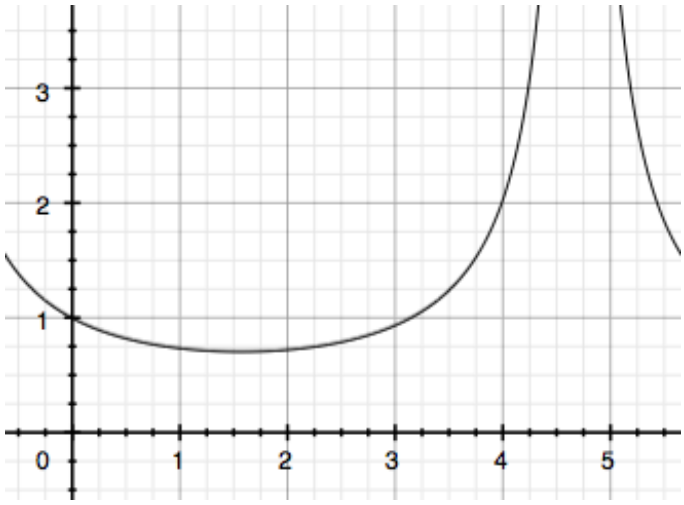
Interval Length	Approximation
0.2	2.8436
0.1	2.8366



Summary:

The calculations for the Trapezoidal Method are easy when the intervals all have the same length.

Shorter intervals usually give better approximations.

Trapezoidal Method calculates the area under a straight-line approximation.